Mereology of thoughts,
and why you can't (statively) think a question

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1 Basic and stative meanings of 'think'

- **Assumption:** both stative and process readings of 'think' are derived from a common (episodic) core meaning.

$$\text{basic meaning} \swarrow \quad \searrow$$

stative think   process think

(1) $[\![x \ \sqrt{\mathit{think}} \ P]\!]^s = 1$ iff $\exists t, s' \in s$ [thought$(t) \wedge \textsc{have}(s')(t)(x) \wedge \textsc{Cont}(t) = P$]

- "There is a thought t, and it is had by x, and its content is P".
- **Stative 'think'** is derived from the basic meaning via the genericity operator GEN:

(2) $[\![\textsc{gen} \ x \ \sqrt{\mathit{think}} \ P]\!]^s = 1$ iff
$\textsc{gen}\ s\ [C(s)(x)]\ \ \exists t, s' \in s$ [thought$(t) \wedge \textsc{have}(s')(t)(x) \wedge \textsc{Cont}(t) = P$]

- "In all contextually restricted situations $C(s)(x)$, there is a thought t with content P that the attitude holder x has."

<u>What kinds of situations are in C?</u>

- $C(s)(x)$ contains those situations in worlds maximally similar to ours where the felicity conditions for x having a thought are met and there are no inhibiting factors.
- (Chierchia 1995): "Perhaps, the only restriction we want to impose...is that <*the attitude holder*> be part of it.":

(3) $[\![\textsc{gen} \ x \ \sqrt{\mathit{think}} \ P]\!]^s = 1$ iff
$\textsc{gen}\ s\ [x \sqsubseteq s]\ \ \exists t, s' \in s$ [thought$(t) \wedge \textsc{have}(s')(t)(x) \wedge \textsc{Cont}(t) = P$]

2 Mereologies of thoughts and their contents

2.1 Structuring thoughts

- At any given moment τ in situation s, an attitude holder x holds a plurality of thoughts T—thoughts that x is having at τ. These are not necessarily beliefs! Just thoughts.

(4) $T_{x,s,\tau} :=_{def} \{t \mid \exists s' \in s \text{ at } \tau[\textsc{have}(s')(t)(x)]\}$

- They are ordered by the parthood relationship $\sqsubseteq$, and there is a maximal element in T, which I'll call $\textsc{sum}_T$. Thoughts form a mereology:

(5) **Axioms of classical extensional mereology**

a. **Reflexivity:** $\forall x, x \sqsubseteq x$
b. **Transitivity:** $\forall x, y, z, (x \sqsubseteq y \wedge y \sqsubseteq z) \rightarrow x \sqsubseteq z$
c. **Antisymmetry:** $\forall x, y(x \sqsubseteq y \wedge y \sqsubseteq x) \rightarrow x = y$

(6) **Overlap:** x ∘ y $\Leftrightarrow \exists z[z \sqsubseteq x \wedge z \sqsubseteq y]$

(7) **Sum** (Tarksi 1929): SUM of a set T is the element s.t.
$\forall y[T(y) \rightarrow y \sqsubseteq \textsc{sum}] \wedge \forall z[z \sqsubseteq \textsc{sum} \rightarrow \exists z'[T(z') \wedge z \circ z']]$
"a sum of the set T is a thing that consists of everything in T and whose parts each overlap with something in T"

2.2 Structuring contents

- Thoughts are entities that have content associated with them.

(8) CONT is a partial function from $D_e \rightarrow D_{cont}$,
where $D_{cont} \subset (D_{st,t} - \varnothing)$ s.t. $\forall C \in D_{cont}$
$\forall p, q \in C[p \neq q \rightarrow [p \cap q = \varnothing \wedge p \cup q \notin C]]$.

- Two types of sets of propositions in D_{cont}:

(9) **Declarative Content {p}**
x has declarative content if CONT(x) is a singleton set

(10) **Inquisitive Content {p, q,...}**
x has inquisitive content if CONT(x) is a set
containing at least two distinct propositions

- Possible $C \in D_{cont}$: (i) singleton sets $\{p\}$, (ii) partitions, e.g. $\{p, \neg p\}$.

- Condition "$p \cup q \notin C$" excludes non-singleton sets containing $\varnothing$ from being in D_{cont}, e.g. $\{p, \varnothing\} \notin D_{cont}$.

(11) **Abbreviation:**
$a = \{w : \text{Ann came in } w\}$, $\neg a = \{w : \text{Ann didn't come in } w\}$,
$b = \{w : \text{Bill came in } w\}$, $\neg b = \{w : \text{Bill didn't come in } w\}$

(12) a. $[\![\text{that Ann came}]\!] = \lambda x.\textsc{cont}(x) = \{a\}$
b. $[\![\text{whether Ann came}]\!] = \lambda x.\textsc{cont}(x) = \{a, \neg a\}$
c. $[\![\text{who came}]\!] = \lambda x.\textsc{cont}(x) = \{ab, a\neg b, \neg ab, \neg a\neg b\}$

- I adopt *"Conjunctive Parthood"* from (Bondarenko & Elliott 2024):

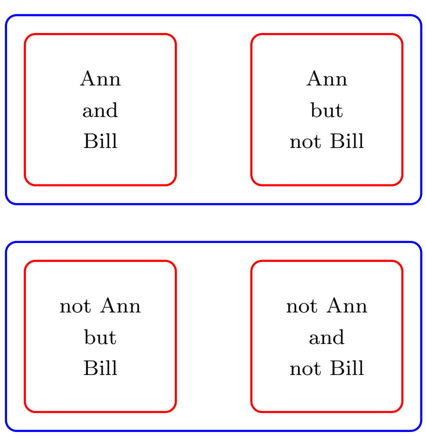
(13) **Conjunctive Parthood** (def.)
For any $P, Q \in D_{st,t}$, $P \sqsubseteq Q$ iff
$\forall p \in P\ \exists q \in Q[q \subseteq p] \quad \wedge \quad \forall q \in Q\ \exists p \in P[q \subseteq p]$

- **NB:** Since we're looking at contents, we will actually only be ordering the sets of propositions in D_{cont}, which satisfy the conditions in (8). Some illustrations:

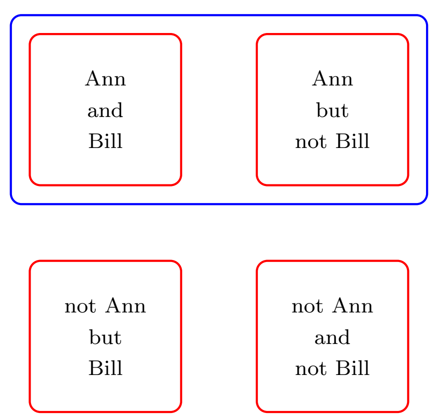
(14) a. $\{\lambda w.$ it is raining in $w\} \sqsubseteq \{\lambda w.$ it is raining heavily in $w\}$
b. The thought is that it's raining heavily.
c. $\rightsquigarrow$Part of this thought is that it's raining.

(15) a. $\{\lambda w.$ it is raining in $w\} \not\sqsubseteq \{\lambda w.$ it is raining in w, $\lambda w.$ it is not raining in $w\}$
b. The question is whether it's raining or not.
c. $\not\rightsquigarrow$Part of this question is that it's raining.

(16) a. $\{\lambda w.$ Ann came in w, $\lambda w.$ Ann didn't come in $w\} \sqsubseteq$
$\{\lambda w.$ Ann and Bill came in w,
$\lambda w.$ Ann didn't come and Bill came in w,
$\lambda w.$ Ann didn't come and Bill didn't come in w,
$\lambda w.$ Ann came and Bill didn't come in $w\}$
b. The question is who came to the party.
c. $\rightsquigarrow$Part of this question is whether Ann came to the party.
d. $\not\rightsquigarrow$Part of this question is that Ann came to the party.



Did Ann come to the party? $\sqsubseteq$ Who came to the party?



Ann came to the party $\not\sqsubseteq$ Who came to the party?
Ann came to the party $\sqsubseteq$ Ann and Bill came to the party

- I will assume that **overlap** and **sum** can be defined for this definition of parthood in a parallel way ($P, Q, R \in D_{cont}$).

(17) **Overlap:** $P \circ Q \Leftrightarrow \exists R[R \sqsubseteq P \wedge R \sqsubseteq Q]$

(18) **Sum** (Tarksi 1929): SUM of a set C is the element s.t.
$\forall P[C(P) \rightarrow P \sqsubseteq \textsc{sum}] \wedge \forall R[R \sqsubseteq \textsc{sum} \rightarrow \exists R'[C(R') \wedge R \circ R']]$

"a sum of the set of contents C is a thing that consists of everything in C and whose parts each overlap with something in C"

- This structure will form a mereology **due to the restrictions we placed on** D_{Cont}.

Reflexivity $\forall P, P \sqsubseteq P$

- Imagine this was false. Then this would need to be false:

(19) $\forall p \in P \; \exists q \in P[q \subseteq p]$

1. $\{\varnothing\} \sqsubseteq \{\varnothing\}$, because $\varnothing$ is a subset of any set.
2. If P contains one or more non-empty propositions: this can't be false because $\forall p[p \subseteq p]$.

Transitivity $\forall P, Q, R \, (P \sqsubseteq Q \wedge Q \sqsubseteq R) \rightarrow P \sqsubseteq R$

- Imagine this was false. Then this would need to hold:

1. **True:** $\forall p \in P \; \exists q \in Q[q \subseteq p] \quad \wedge \quad \forall q \in Q \; \exists p \in P[q \subseteq p]$
2. **True:** $\forall q \in Q \; \exists r \in R[r \subseteq q] \quad \wedge \quad \forall r \in R \; \exists q \in Q[r \subseteq q]$
3. **False:** $\forall p \in P \; \exists r \in R[r \subseteq p] \quad \wedge \quad \forall r \in R \; \exists p \in P[r \subseteq p]$

- The conjunction is false if one of the conjuncts is. Assume the first conjunct is false:

(20) $\exists p \in P \; \neg\exists r \in R[r \subseteq p]$
Let's instantiate this $p \in P$ as p_1. Because $P \sqsubseteq Q$, we get that $\exists q \in Q[q \subseteq p_1]$. Let's instantiate it as q_1. Because $Q \sqsubseteq R$, $\exists r \in R[r \subseteq q_1]$. Let's instantiate it as r_1. Due to transitivity of subsethood, we get that $r_1 \subseteq p_1$, arriving at a contradiction.

- Assume the second conjunct is false:

(21) $\exists r \in R \; \neg\exists p \in P[r \subseteq p]$
Let's instantiate this $r \in R$ as r_1. Because $Q \sqsubseteq R$, we get that $\exists q \in Q[r_1 \subseteq q]$. Let's instantiate it as q_1. Because $P \sqsubseteq Q$, $\exists p \in P[q_1 \subseteq p]$. Let's instantiate it as p_1. Due to transitivity of subsethood, we get that $r_1 \subseteq p_1$, arriving at a contradiction.

Anti-symmetry $\underline{\forall P, Q\ (P \sqsubseteq Q \wedge Q \sqsubseteq P) \rightarrow P = Q}$

- Imagine this was false. Then this would need to hold:

1. **True:** $\forall p \in P\ \exists q \in Q[q \subseteq p] \quad \wedge \quad \forall q \in Q\ \exists p \in P[q \subseteq p]$
2. **True:** $\forall q \in Q\ \exists p \in P[p \subseteq q] \quad \wedge \quad \forall p \in P\ \exists q \in Q[p \subseteq q]$
3. $\underline{\exists p[p \in P \wedge p \notin Q]}$

- If P and Q are not identical, there must be a proposition that is in one of them but not in the other. Let's instantiate this proposition as p.

- Because $P \sqsubseteq Q$, it follows that $\exists q \in Q[q \subseteq p]$. Let's instantiate it as q_1 Because $Q \sqsubseteq P$, it follows that $\exists q \in Q[p \subseteq q]$. Let's instantiate it as q_2. Since by assumption $p \notin Q$, it follows that $p \neq q_1$ and $p \neq q_2$. Thus, we get that $\underline{[q_1 \subset p] \wedge [p \subset q_2]}$, which means that $\underline{q_1 \subset q_2}$.

(22) $\underline{\mathbf{q1} \subset \mathbf{q2}}$
   a. $\underline{\text{Imagine } q_1 \neq \varnothing \wedge q_2 \neq \varnothing.}$
   Due to restrictions on content, we arrive at the contradiction: "$q_1 \cap q_2 = \varnothing$" is incompatible with "$q_1 \cap q_2 = q_1$" if $q_1 \neq \varnothing$.
   b. $\underline{\text{Imagine } q_2 = \varnothing}$. Then we get that $p \subset \varnothing$, from which it would follow that $p = \varnothing$. But this then violates our assumption that $p \in P \wedge p \notin Q$.
   c. $\underline{\text{Imagine } q_1 = \varnothing,\ q_2 \neq \varnothing}$. Then we arrive at a contradiction due to the definition of D_{cont}. If $Q \in D_{cont}$, then for any two distinct propositions in Q it must be the case that their union is not in Q. However, $(q_1 \cup q_2) = (\varnothing \cup q_2) = q_2$, and $q_2 \in Q$. Thus, it must be the case that $P = Q$.

- **NB:** Restrictions on the set of propositions that Cont introduces is crucial to the structure of contents having the anti-symmetry property.

- A contradictory thought state can be represented as having content $\{\varnothing\}$. Having two thoughts with contradictory contents results in the $\textsc{sum}_C$ being contradictory:

(23)
   a. If $\{p\} \sqsubseteq \textsc{sum}_C$, $\forall q \in \textsc{sum}_C[q \subseteq p]$
   b. If $\{\neg p\} \sqsubseteq \textsc{sum}_C$, $\forall q \in \textsc{sum}_C[q \subseteq \neg p]$
   c. The only way for these to hold at the same time is if $\textsc{sum}_C = \{\varnothing\}$.

3 Mappings between thoughts and contents

- This is the principle that we will need: it ensures that content of a bigger thought has content of a smaller thought as its part.

(24) *Mapping to SuperContents*
   ∀ t,t' ∈ D_e, P' ∈ D_{cont}:
   Cont(t') = P' $\wedge\ t' \sqsubseteq t \rightarrow \exists P[P' \sqsubseteq P \wedge \textsc{Cont}(t) = P]$

- Here are two other principles that we might want to have:

(25) *Mapping to SubContents*
$\forall$ t,t' $\in$ D_e, P $\in$ D_{cont}:
$\textsc{Cont}(\mathrm{t}) = \mathrm{P} \wedge t' \sqsubseteq t \rightarrow \exists P'[P' \sqsubseteq P \wedge \textsc{Cont}(t') = P']$

(26) *Mapping to SubEntities*
$\forall P, P' \in D_{cont}, \forall t \in D_e$
$\textsc{Cont}(t) = P \wedge P' \sqsubseteq P \rightarrow \exists t'[t' \sqsubseteq t \wedge Cont(t') = P']$

- *Mapping to SubContents* ensures that content of smaller thoughts is part of the content of bigger thoughts.

- *Mapping to SubEntities* ensures that for each part of the content of a thought t, there is a corresponding thought-part.

- Here's on the other hand is a principle that we probably don't want:

(27) *Mapping to SupEntities*
$\forall P, P' \in D_{cont}, \forall t' \in D_e$
$\textsc{Cont}(t') = P' \wedge P' \sqsubseteq P \rightarrow \exists t[t' \sqsubseteq t \wedge Cont(t) = P]$

- This would make us postulate an infinite amount of bigger thoughts, it seems.

4 Why stative 'think' is incompatible with questions

- As soon as an attitude holder has a thought whose content is inquisitive, the thought-sum must be either inquisitive, or be the contradictory thought.

(28) **Inquisitive Thought $\rightsquigarrow$ Inquisitive Sum or a Contradictory State**

a. $\exists P \sqsubseteq \textsc{sum}_C[\exists p, p' \in P[p \neq p']]$
b. $\forall p, p' \in P[p \neq p' \rightarrow [p \cap p' = \varnothing \wedge p \cup p' \notin P]]$. *because* $P \in D_{cont}$
c. $\forall p \in P \exists q \in \textsc{sum}_C[q \subseteq p] \wedge \forall q \in \textsc{sum}_C \exists p \in P[q \subseteq p]$ *because* $P \sqsubseteq \textsc{sum}_C$
d. Let us assume that $|\textsc{sum}_C| = 1$, and the unique proposition in $\textsc{sum}_C$ is q. Let us take any two distinct propositions p and p' in P.
e. Because $\forall p \in P \exists q \in \textsc{sum}_C[q \subseteq p]$, it must be true that $q \subseteq p \wedge q \subseteq p'$
f. Because the question-thought P is a partition, $p \cap p' = \varnothing$, and so $q = \varnothing$. Thus, the only way for inquisitive thought to be part of a declarative sum is if the sum is $\{\varnothing\}$—*the contradictory thought*.

- Declarative thoughts on the other hand do not force the sum to be declarative:

(29) **Declarative Thought $\not\rightsquigarrow$ Declarative Sum**

a. $\exists P \sqsubseteq \textsc{sum}_C[|P| = 1]$. Let us assume that $P = \{p\}$.
b. $\forall p \in P \exists q \in \textsc{sum}_C[q \subseteq p] \wedge \forall q \in \textsc{sum}_C \exists p \in P[q \subseteq p]$ *because* $P \sqsubseteq \textsc{sum}_C$
c. $\textsc{sum}_C$ can be inquisitive as long as: $\forall q \in \textsc{sum}_C[q \subseteq p]$

d. For example, we could have a small plurality consisting of just three thoughts t_1, t_2 and the SUM_T:

- $\bullet\textsc{Cont}(t_1) = \{p\}$
- $\bullet\textsc{Cont}(t_2) = \{q, \neg q\}$
- $\bullet\textsc{Cont}(\text{SUM}_T) = \text{SUM}_C = \{p \cap q, p \cap \neg q\}$
- $\bullet\{p\} \sqsubseteq \{p \cap q, p \cap \neg q\}$
- $\bullet\{q, \neg q\} \sqsubseteq \{p \cap q, p \cap \neg q\}$

- Recall that we're considering stative 'think', created by the use of the genericity operator with a very wide restrictor: $C(s)(x)$ will contain all situations which x is part of.
- This implies that $C(s)(x)$ will contain both situations with declarative thought sums, including non-contradictory ones, and situations with inquisitive thought sums, (31).

(30) $[\![\text{GEN } x \ \sqrt{think}\ \ P]\!]^s = 1$ iff
$\text{GEN } s\ \ [C(s)(x)]\ \ \exists t, s' \in s\ [\text{thought}(t) \wedge \text{HAVE}(s')(t)(x) \wedge \textsc{Cont}(t) = P]$

(31) **Diversity of GEN $\sqrt{think}$**
$\exists s' \in C(s)(x)[|\text{CONT}(\text{SUM}_{Tx,s',\tau})| = 1] \wedge \exists s' \in C(s)(x)[|\text{CONT}(\text{SUM}_{Tx,s',\tau})| > 1]$, and
$\exists s' \in C(s)(x)[|\text{CONT}(\text{SUM}_{Tx,s',\tau})| = 1 \wedge \text{CONT}(\text{SUM}_{Tx,s',\tau}) \neq \varnothing]$

- Thus, the meaning of GEN $\sqrt{think}$ is incompatible with P being inquisitive:

(32) $\{s \mid \exists t, s' \in s\ [\text{thought}(t) \wedge \text{HAVE}(s')(t)(x) \wedge |\textsc{Cont}(t)| \geq 1]\} \subset \{s \mid C(s)(x)\}$

is incompatible with

$\{s \mid C(s)(x)\} \subseteq \{s \mid \exists t, s' \in s\ [\text{thought}(t) \wedge \text{HAVE}(s')(t)(x) \wedge |\textsc{Cont}(t)| \geq 1]\}$

- Hence, stative 'think' will never be compatible with questions.
- Dynamic 'think' is compatible with questions because it does not involve generic quantification. We can think of this meaning as arising due to the DO operator, which creates a complex event consisting of multiple having-thought events with content P:

(33) $[\![\text{DO } x\ \sqrt{think}\ \ P]\!]^s = 1$ iff
$\exists e, e', e'' \in s\ [e' \neq e'' \wedge e' \sqsubseteq e \wedge e'' \sqsubseteq e \wedge \forall e' \sqsubseteq e$
$[\exists t \in s :\ \text{thought}(t) \wedge \text{HAVE}(e')(t)(x) \wedge \textsc{Cont}(t) = P]]$

- **Desirable consequence:** with questions, process 'think' will imply that $T_{x,s,\tau}$ for any τ at which a subevent of the thinking event holds would contain only inquisitive thought-sums. This gives us an attitude holder wondering P.